Exercise 3.3.1. Find a basis for the subspace of $\mathbb{R}^4$ spanned by the vectors $(1, 2, -1, 0)$, $(4, 8, -4, -3)$, $(0, 1, 3, 4)$, $(2, 5, 1, 4)$.

Solution. $\{(1, 2, -1, 0), (4, 8, -4, -3), (0, 1, 3, 4)\}$ is a basis.

Exercise 3.3.5. Find a basis for the space of symmetric $n \times n$ matrices.

Solution. $\left\{A^{(i,j)} : i \leq j\right\}$, where $A^{(i,j)}_{i,j} = A^{(i,j)}_{j,i} = 1$ with remaining elements equal to 0.

Exercise 3.3.6. Prove that a square matrix A is invertible if and only if its columns are linearly independent.

Proof. If A is invertible, $Ax = 0$ has only one trivial solution $x = 0$, which says

$$x_1 \cdot a_1 + x_2 \cdot a_2 + \ldots + x_n \cdot a_n = 0.$$

□

Exercise 3.3.7. Let V be the vector space of functions on the interval $[0, 1]$. Prove that the functions $x^3, \sin x$ and $\cos x$ are linearly independent.

Proof. Evaluate 3 points at $0, \frac{\pi}{6}, \frac{\pi}{4}$.

□